Treatment Effect Heterogeneity

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Direct Analysis of Heterogeneity in Treatment Effects

Essential Heterogeneity

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Direct Analysis of Heterogeneity in Treatment Effect

- Pros
  - Analysis of heterogeneity of treatment effects via regression is straightforward and policy relevant
  - $Y_i = \beta X_i + \gamma T_i + \delta(X_i \times T_i) + \varepsilon_i$
  - $\widehat{\gamma}$ gives the impact of the treatment with $X = 0$
  - $\widehat{\gamma}$ gives the differential marginal impact of the treatment as you increase X

Direct Analysis of Heterogeneity in Treatment Effect

- Cons:
  - The Analysis of Heterogeneity is the last refuge of the insignificant or underpowered RCT; look at enough covariates and you are sure to find heterogeneity on something
  - Pre-analysis plans have taken on a particularly important role for the analysis of heterogeneity; also the use of blocking or stratification in the research design to signal the covariates over which you plan to look for differential treatment effects
  - Interpretation: $X's$ not randomly assigned

Direct Analysis of Heterogeneity in Treatment Effect

- Critical to use multiple inference corrections:
  - Bonferroni: if the p-value for rejection in a single hypothesis test is α (.05), then with q tests performed the rejection statistic should become $\frac{\alpha}{q}$
  - False Discovery Rate (Anderson 2008, JASA). Provides Stata code to correct p-values for the number of hypotheses tested
  - Use indexes or the Mean Effects techniques of Kling, Liebman, and Katz (2007) to account for the covariance between interaction variables

Direct Analysis of Heterogeneity in Treatment Effects

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Essential Heterogeneity

- Papers by Heckman, Vytlacil, and co-authors introduced two important new ideas into the conceptualization and analysis of treatment effects (not only in RCTs!)

  1. **Essential Heterogeneity**: Not only do individuals have heterogeneous treatment effects, but they partially understand this heterogeneity. Hence the rate at which they comply with the treatment is a function of their (unobserved) treatment effect

  2. **Marginal Treatment Effects**: Conditional impact of treating an individual on a set of observables and for a given (potentially unobserved) propensity to comply with the treatment. The MTE allows for the unification of numerous different types of treatment effects within a single structure

Essential Heterogeneity

- Imagine that potential outcomes can be written as:

$$\begin{aligned} Y_1 &= X'\beta^1 + U^1 \\ Y_0 &= X'\beta^0 + U^0 \end{aligned}$$

- X is a vector of observable attributes
- U is an unobserved residual
- The treatment effect is $\Delta = Y_1 - Y_0 = X'(\beta^1 - \beta^0) + U^1 - U^0$

Essential Heterogeneity

- Let $D(Z)$ denote the observed treatment decision
- Let $D^*(Z)$ denote latent variable that generates $D(Z)$

$$
\begin{aligned}
D^* &= Z'\theta + U^D \\
D(Z) &= \mathbb{1}_{D^*(Z)\geq 0} = \mathbb{1}_{Z'\theta + U^D \geq 0}
\end{aligned}
$$

- Exclusion restriction (i.e., some element of Z which is not contained in X)
- By varying Z, manipulate probability of receiving treatment without affecting potential outcomes
- Assume (U^D, U^I, U^0) is independent of X and Z

Essential Heterogeneity

- Assumptions in more complicated (non-linear) models

1. The term $\mu_D(Z)$ is a nondegenerate random variable conditional on X (i.e., Z has independent predictive power on compliance above and beyond X or Z is a relevant instrument for compliance)

2. (U_1, U_c) and (U_0, U_c) are independent of Z conditional on X (i.e., Z is a valid instrument for compliance)

3. The distribution of $\mu_D(Z)$ is absolutely continuous with respect to Lebesgue measure (convenient for derivation/estimation)

4. $\sup_v E(|Y_1||U = u) < \infty$, $\sup_v E(|Y_0||U = u) < \infty$; (Potential outcomes are finite)

5. $0 < Pr(D = 1) < 1$ (compliance probabilities strictly between 0 and 1)

Marginal treatment effect (MTE)

Remember that $\Delta = Y_1 - Y_0 = X'(\beta^1 - \beta^0) + U^1 - U^0$

- Important building block is the Marginal treatment effect (MTE):

$$\begin{aligned}
\Delta^{MTE}(x, u^d) &= E(\Delta | X = x, U^d = u^D) \\
&= x'(\beta^1 - \beta^0) + E(U^1 - U^0 | U^D = u^D, X = x) \\
&= x'(\beta^1 - \beta^0) + E(U^1 - U^0 | U^D = u^D)
\end{aligned}$$

- Evaluation of the MTE parameter at low values of u^D averages the outcome gain with unobservables making them least likely to participate

- Evaluation of parameter at high values of u^D is the gain for those individuals with unobservables them most likely to participate

Local Average Treatment Effect (LATE)

- LATE of Imbens and Angrist (1994) estimates the expected gain for those induced to receive treatment through a change in the instrument from $Z = z$ to $Z = z'$

$$
\begin{aligned}
\Delta^{LATE}(x, u'_d, u_d) &= E(\Delta | X = x, D(z) = 0, D(z') = 1) \\
&= x(\beta^1 - \beta^0) + E(U^1 - U^0 | -z'\theta \leq U^D \leq z\theta, X = x) \\
&= x(\beta^1 - \beta^0) + E(U^1 - U^0 | -z'\theta \leq U^D \leq z\theta) \\
&= \frac{1}{u_d - u'_d} \int_{u_d}^{u'_d} \Delta^{MTE}(x, u) du
\end{aligned}
$$

Local Average Treatment Effect (LATE)

- Then, as Z' and Z become arbitrarily close

$$\lim_{u_d' \to u_d} \Delta^{LATE}(x, u_d', u_d) = \Delta^{MTE}(x, u_d)$$

- LATE measures the average MTE across a range of the unobserved selection distribution

- As that range converges to zero the LATE converges to the MTE evaluated exactly at a single point on the distribution

Average Treatment effect (ATE)

This structure allows us to unify a variety of treatment effects as follows:

$$
\begin{aligned}
ATE(x) &= E(\Delta | X = x) \\
&= x(\beta^1 - \beta^0) + E(U^1 - U^0 |, X = x) \\
&= x(\beta^1 - \beta^0)
\end{aligned}
$$

$$\int_0^1 \Delta^{MTE}(x, u) du$$

Treatment on the treated (TOT)

$$
\begin{aligned}
D(z) = 1) &= E(\Delta | X = x, Z = z, D(z) = 1) \\
&= x(\beta^1 - \beta^0) + E(U^1 - U^0 | U^D \geq -z'\theta) \\
&= \int_0^1 \Delta^{MTE}(x, u) h_{ToT} du
\end{aligned}
$$

- h_{ToT} is the inverse of the compliance rate induced by the experiment

Policy Relevant Treatment Effect

- Heckman and Vytlacil also introduce the Policy Relevant Treatment Effect: mean effect of going from a baseline policy to an alternative policy per net person shifted

$$\frac{E(Y') - E(Y)}{E(D') - E(D)}$$

where the prime refers to an alternate policy

Marginal treatment effect

Several conceptual extensions to the Essential Heterogeneity concept:

- Variety of researchers have worked to build auction or WTP revelation mechanisms into experiments
  - Under perfect information the willingness to pay to receive the treatment should be a direct measure of the MTE
  - Examples of this include the Becker-Degroot-Marschak (BDM) mechanism, and more recently the 'Take it or Leave it' (TIOLI) pricing experiments of Chassang, Padro i Miguel and Snowberg
- The entire essential heterogeneity framework is still assuming that the treatment effects themselves are invariant
  - Identified only under the assumption that treatments alter ONLY the propensity to enter the treatment
  - But but not the impact of the treatment itself. This is clearly not the case for many interventions